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The Assessment of the Temporal Evolution of Space Geodetic Terrestrial Reference Frames

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1. Introduction

The assessment of global Terrestrial Reference Frames (TRFs) is based on the well known Helmert type transformation. For example, the assessment of the origin and its variation in time (rate) of a TRF is realized through the comparison of the Helmert translation parameters between the SLR-only and the TRF based on all space geodetic techniques. (Altamimi et al. 2011, Seitz et al. 2012, Altamimi and Dermanis 2012). In addition, the scale and its rate is evaluated through the comparison of the SLR/VLBI-only TRF. In the case of the orientation (spatial and time-dependent), none of the techniques can provide any information. A main question on the assessment of Helmert type transformation is if there are some systematics that cannot be detected. We shall present an alternative approach on the assessment of the temporal evolution (only wrt the time dependent part) of the TRF, here taking DTRF2008 (Seitz et al. 2012) as an example. We prove that the new methodology finds some systematic effects which are embedded to each of the four space techniques specific TRFs.

2. The mathematical model

Let us assume that the velocity v of any point i can be divided into two parts: (a) The "unreal" quantity v^{tr} which refers to the datum effect and is directly connected to the choice of reference frame (e.g constraints) and (b) the deformation part δv which reflects the tectonic motion and post glacial rebound. The deformation term contains also the relatively small random errors. We have :

$$v_i = v_i^{tr} + \delta v_i \qquad (1)$$

The straightforward relation between the coordinate vector x in two different epochs t_0 and t is:

$$x_i(t) = x_i(t_0) + v_i(t - t_0)$$
$$x_i(t) = x_i(t_0) + v_i^{tr}(t - t_0) + \delta v_i(t - t_0) \qquad (2)$$
$$x_i(t) = x_i(t_0) + \delta x_i(t) + \delta v_i(t - t_0)$$

The term $\delta x(t)$ corresponds to a change between the two epochs and it can be expressed by the Helmert transformation as follows:

$$\delta x_i(t) = v_i^{tr}(t - t_0) = E_i \theta(t) \qquad (3)$$

where E_i and are the design matrix and $\theta(t)$ the 7 Helmert parameters (translations, scale and orientations)

Combining Eqs. (2) and (3) we finally get:

$$x_i(t) - x_i(t_0) = E_i \theta(t) + \delta v_i(t - t_0) \qquad (4)$$

After dividing with the time difference term $t-t_0$ and since $\frac{x_i(t) - x_i(t_0)}{t - t_0} = v_i$ we get:

$$v_i = E_i \frac{\theta(t)}{t - t_0} + \delta v_i \qquad (5)$$

For all the network points, Eq.(5) yields:

$$v = E \dot{\theta} + \delta v \qquad (6)$$

Applying the LS principle we estimate the parameter rates:

$$\dot{\theta} = (E^T C^{-1} E)^{-1} E^T C^{-1} v \qquad (7)$$

where C is the error covariance matrix of the observed velocities. Eq. (7) can be used an assessment tool for the temporal evaluation of the TRF in a <u>relative sense</u> (= differences between two frames or two different realizations). Since the associated rates are strongly correlated with the reference frame choice, this approach can be used for the comparison of

1.Different TRFs

2.The same TRF using different datum choices (minimal constraints)

3.The comparison of a space-wise TRF versus the combined solution

3. The relation with the Helmert Transformation

The differences between the rates of two frames can be used as a quality measure for TRF assessment. The well known Helmert transformation is the main tool for this. The two approaches, new and Helmert coincide **if and only if**:

1. They are applied to common points.
2. They share a common error covariance matrix of the observations (common weight matrix).

If these conditions are fulfilled but the numerical results of the two methodologies differ, it is rather possible that <u>some relative biases exist affecting the results</u> which are not detectable when the Helmert transformation is applied. The quantities we compare are:

(a) The rate differences between two frames, using the new methodology with
(b) the classical Helmert transformation rate parameters

4. Numerical application: The DTRF2008 technique-wise TRFs

For the numerical tests we use the associated technique-wise TRFs of the DTRF2008. We compare each space technique frame with the combined solution DTRF2008. We implement our methodology to the stations of VLBI, SLR, GPS and DORIS of their own TRFs and then to the same stations wrt the DTRF 2008. The error covariance matrices practically coincide in the case of the technique-wise TRFs and the final combined solution.

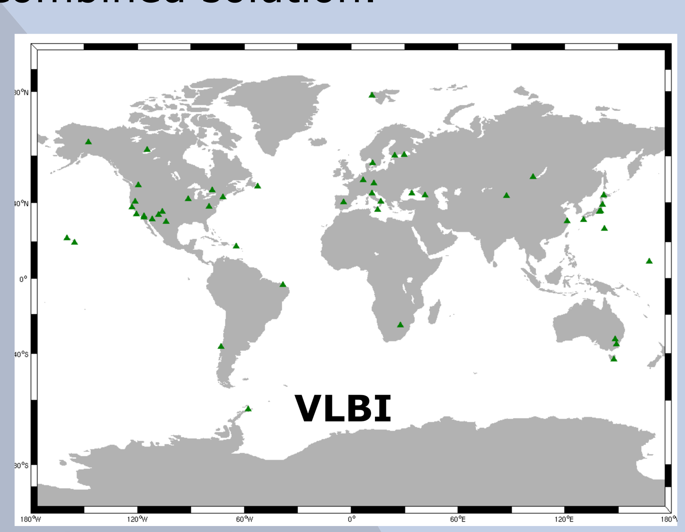

Figure 1: The DTRF2008 VLBI network.

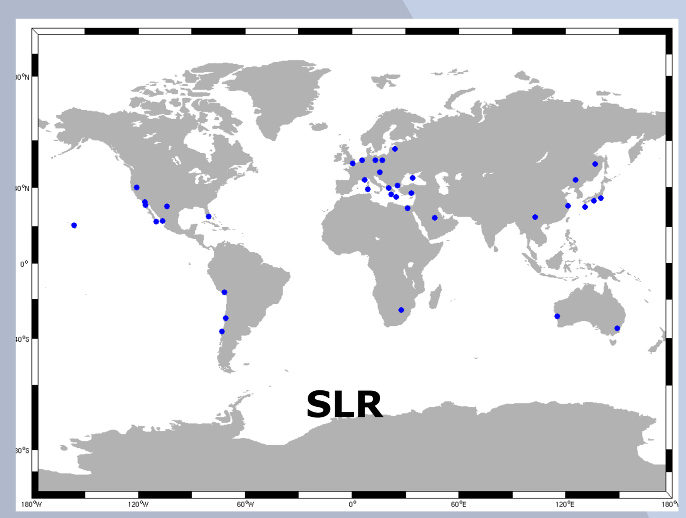

Figure 2: The DTRF2008 SLR network.

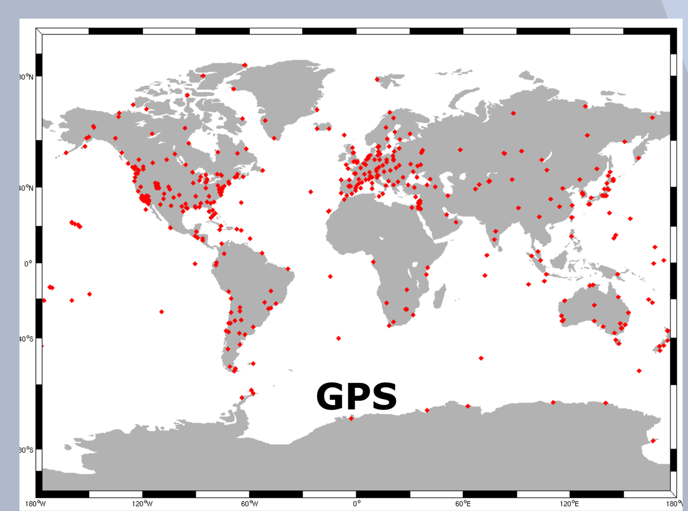

Figure 3: The DTRF2008 GPS network.

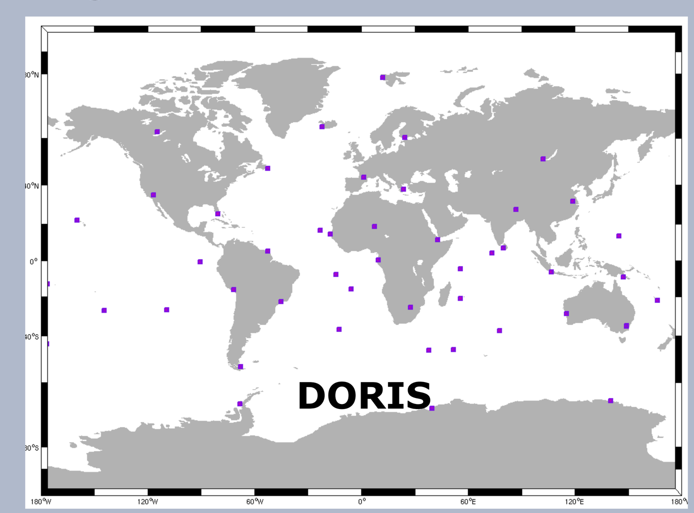

Figure 4: The DTRF2008 DORIS network.

Table 1: The comparison of the two methodologies for the VLBI network.

parameters*	New approach	Helmert trans.	differences
t_x dot	-0.41±0.04	-0.35±0.11	-0.06±0.11
t_y dot	-0.29±0.03	-0.35±0.11	0.06±0.11
t_z dot	-0.10±0.04	0.02±0.11	-0.12±0.11
ds dot	-0.03±0.03	-0.05±0.07	0.02±0.08
r_x dot	1.12±0.06	1.22±0.16	-0.10±0.17
r_y dot	-0.39±0.06	-0.33±0.16	-0.06±0.17
r_z dot	-0.47±0.07	-0.39±0.16	-0.08±0.17

Table 2: The comparison of the two methodologies for the SLR network.

parameters	New approach	Helmert trans.	differences
t_x dot	-0.07±0.04	-0.05±0.10	-0.02±0.11
t_y dot	0.15±0.04	0.12±0.10	0.03±0.11
t_z dot	0.08±0.04	0.06±0.10	0.02±0.11
ds dot	-0.07±0.03	-0.10±0.10	0.03±0.11
r_x dot	-0.09±0.07	0.06±0.12	-0.15±0.14
r_y dot	-0.03±0.07	-0.06±0.12	0.03±0.14
r_z dot	-0.37±0.08	-0.28±0.12	-0.09±0.14

Table 3: The comparison of the two methodologies for the GPS network.

parameters	New approach	Helmert trans.	differences
t_x dot	-0.01±0.08	-0.04±0.10	0.03±0.13
t_y dot	0.11±0.08	0.10±0.10	0.01±0.13
t_z dot	-0.08±0.08	-0.05±0.10	-0.03±0.13
ds dot	0.22±0.08	0.18±0.01	0.04 ±0.08
r_x dot	0.00±0.07	-0.03±0.03	0.03±0.08
r_y dot	0.00±0.07	-0.03±0.03	0.03±0.08
r_z dot	0.03±0.08	0.06±0.03	-0.03±0.09

Table 4: The comparison of the two methodologies for the DORIS network.

parameters	New approach	Helmert trans.	differences
t_x dot	-0.13±0.10	-0.10±0.24	-0.03±0.26
t_y dot	0.32±0.10	0.40±0.24	-0.08±0.26
t_z dot	0.75±0.10	0.80±0.23	-0.05±0.26
ds dot	-0.20±0.08	-0.15±0.24	-0.05±0.25
r_x dot	-0.93±0.08	-0.95±0.31	0.02±0.32
r_y dot	0.18±0.10	0.00±0.31	0.18±0.33
r_z dot	0.13±0.10	0.03±0.31	0.10±0.33

*all the parameters are expressed in mm/yr

5. Conclusions

The results reveal that in all the cases the two methodologies do not differ significantly. The formal errors of the parameters new proposed methodology are smaller than in the case the classical Helmert transformation. However, we found large though not significant differences for the translation rate of the z axis for VLBI and for the orientation rate about the x axis for SLR, respectively. The scale of the SLR and VLBI TRFs does not show any discrepancy. In addition, the SLR translation rates do not seem to carry any systematic effects. We can imply that the two methodologies perform the same results with uncertainty of **0.1 mm/yr** for VLBI, SLR and GPS and **0.3 mm/yr** for DORIS, respectively.

6. References

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